LIMIT LAWS

1. $\lim\limits_{x\to 0}\frac{\sin x}{x}=1$ 2. $\lim\limits_{x\to\infty}\frac{\sin x}{x}=0$ 3. $\lim\limits_{x\to 0}\frac{\sin^2 x}{x}=0$

4. $\lim\limits_{x\to 0}\frac{\cos x-1}{x}=0$ 5. $\lim\limits_{x\to 0}\frac{\sin(ax)}{(bx)}=\lim\limits_{x\to 0}\frac{\sin(ax)}{\sin(bx)}=\lim\limits_{x\to 0}\frac{(bx)}{\sin(ax)}=\frac{a}{b}$

6. $\lim\limits_{x\to a}f(x)=L$ (exists) If and only if $\lim\limits_{x\to a^+}f(x)=\lim\limits_{x\to a^-}f(x)=L$

7. $f(x)$ is cont a if $\lim\limits_{x\to a}f(x)=f(a)$

DIFFERENTIATION RULES

Basic Rules

1. $\frac{d}{dx}c=0$ (Constant) 2. $\frac{d}{dx}c[f(x)]=c\frac{d}{dx}f(x)$ (constant multiple)

3. $\frac{d}{dx}x^n=nx^{n-1}$ (power) 4. $\frac{d}{dx}(u\pm v)=\frac{d}{dx}u\pm\frac{d}{dx}v$ (Sum & Difference)

5. $\frac{d}{dx}(uv)=v\frac{d}{dx}u+u\frac{d}{dx}v$ (Product)

6. $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{d}{dx}u-u\frac{d}{dx}v}{v^2}$ (Quotient)

7. $\frac{d}{dx}f(g(x))=\frac{d}{dx}f(u)\bullet\frac{d}{dx}g(x)$ where $u=g(x)$ (Quotient)

Trigonometric Functions

8. $\frac{d}{dx}\sin u=\cos u\frac{du}{dx}$ 9. $\frac{d}{dx}\cos u=-\sin u\frac{du}{dx}$

10. $\frac{d}{dx}\tan u=\sec^2 u\frac{du}{dx}$ 11. $\frac{d}{dx}\cot u=-\csc^2 u\frac{du}{dx}$

12. $\frac{d}{dx}\sec u=\sec u\tan u\frac{du}{dx}$ 13. $\frac{d}{dx}\csc u=-\csc u\cot u\frac{du}{dx}$

Inverse Trigonometric Functions

14. $\frac{d}{dx}\sin^{-1}u=\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$ 15. $\frac{d}{dx}\cos^{-1}u=\frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$

16. $\frac{d}{dx}\tan^{-1}u=\frac{1}{1+u^2}\frac{du}{dx}$ 17. $\frac{d}{dx}\cot^{-1}u=\frac{-1}{1+u^2}\frac{du}{dx}$

18. $\frac{d}{dx}\sec^{-1}u=\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$ 19. $\frac{d}{dx}\csc^{-1}u=\frac{-1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$

Exponential and Logarithmic Functions

20. $\frac{d}{dx}\ln u=\frac{1}{u}\frac{du}{dx}$ 21. $\frac{d}{dx}\log_a u=\frac{1}{u\ln a}\frac{du}{dx}$

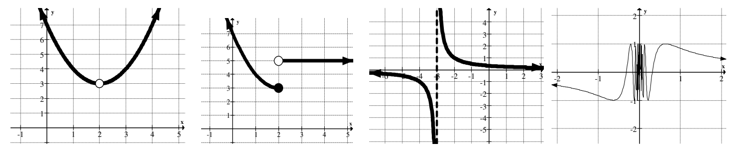
22. $\frac{d}{dx}e^u=e^u\frac{du}{dx}$ 23. $\frac{d}{dx}a^u=a^u\ln a\frac{du}{dx}$

24. $\frac{d}{dx}\sqrt{u}=\frac{1}{2\sqrt{u}}\frac{du}{dx}$ 25. $\frac{d}{dx}|u|=\frac{u}{|u|}\frac{du}{dx}$

ARITHMETIC OF INFINITY

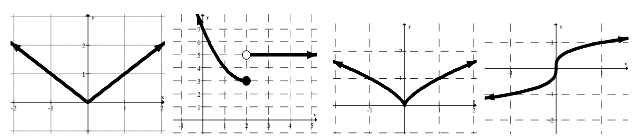
Sum 1. $\infty+\infty=\infty$ 2. $n+\infty=\infty$ 3. $\infty+0=\infty$ 4. $0+\infty=\infty$	**Product** 1. $\infty\bullet\infty=\infty$ 2. $n\bullet\infty=\infty\bullet n=\infty$ 3. $0\bullet\infty=\infty\bullet 0=0$
Difference 1. $\infty-\infty=und$ 2. $n-\infty=-\infty$ 3. $\infty-n=\infty$ 4. $n-n^-=0^-=-\frac{1}{\infty}$ 5. $n^+-n=0^+=\frac{1}{\infty}$	**Quotient** 1. $\infty/\infty=und$ 2. $n/\pm\infty=0$ 3. $\infty/n=\infty$ 4. $n/0=\pm\infty$ 5. $n/0^+=\infty$ 6. $n/0^-=-\infty$
Power 1. $\infty^\infty=\infty$ 2. $\infty^n=\infty$ 3. $\infty^0=1$ 5. $0^\infty=0$ 4. $n^\infty=\infty$	**Trig** 1. $-1\le\sin(\pm\infty)\le 1$ 2. $-1\le\cos(\pm\infty)\le 1$

TYPES OF DISCONTINUITY



Point or removable Jump Infinite Oscillating

WHERE THE DERIVATIVE DOES NOT EXIST



Corner Discontinuity Cusp Vertical tangent

ANTIDIFFERENTION (INTEGRATION) RULES

1. $\int x^n dx=\frac{x^{n+1}}{n+1}+C$ 2. $\int\frac{1}{x}dx=\ln|x|+C$ 3. $\int e^{kx}dx=\frac{e^{kx}}{k}+C$

4. $\int\sin kx\,dx=-\frac{\cos kx}{k}+C$ 5. $\int\cos kx\,dx=\frac{\sin kx}{k}+C$ 6. $\int\sec x\tan x\,dx=\sec x+C$

7. $\int\sec^2 x\,dx=\tan x+C$ 8. $\int\csc x\cot x\,dx=-\csc x+C$ 9. $\int a^x dx=\frac{a^u}{\ln a}+C$

Definition of Derivative

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$f'(x)\approx$ **Average rate of change**

= Slope of Secant line

$$=\frac{f(b)-f(a)}{b-a}$$

FTC I: $\int_a^b f'(x)dx=f(b)-f(a)$

FTC II i) $\int_a^x f(t)dt=F(x)$

ii) $\frac{d}{dx}\int_a^x f(t)dt=f(x)$ iii) $\frac{d}{dx}\int_a^{g(x)}f(t)dt=f(g(x))\cdot g'(x)$

APPROXIMATING AREA

LRAM$_n=w(f(x_1)+f(x_2)+..+f(x_{n-1}))$ **or** $w_1f(x_1)+w_2f(x_2)+..+w_{n-1}f(x_{n-1})$

RRAM$_n=w(f(x_2)+f(x_3)+\cdots+f(x_n))$ **or** $w_1f(x_2)+w_2f(x_2)+..+w_{n-1}f(x_n)$

MRAM$_n=w\left(f\left(\frac{x_1+x_2}{2}\right)+f\left(\frac{x_2+x_3}{2}\right)+\cdots+f\left(\frac{x_{n-1}+x_n}{2}\right)\right)$ *or*

$$w_1f\left(\frac{x_1+x_2}{2}\right)+w_2f\left(\frac{x_2+x_3}{2}\right)+\cdots+w_{n-1}f\left(\frac{x_{n-1}+x_n}{2}\right)$$

Note: $w=\frac{b-a}{n}$ **and applies only for equal sub intervals**

T$_n=\frac{w}{2}(y_1+2y_2+\cdots+2y_{n-1}+y_n)$ **or** $\frac{1}{2}(w_1(y_1+y_2)+w_2(y_2+y_3)+...)$

Mean Value Theorem: If f is cont on $[a,b]$ and diff on (a,b)

$\Rightarrow$ exist a $c\in(a,b)$ s.t. $f'(c)=\frac{f(b)-f(a)}{b-a}$

Rolle's Theorem: MVT where $f'(c)=\frac{f(b)-f(a)}{b-a}=0$

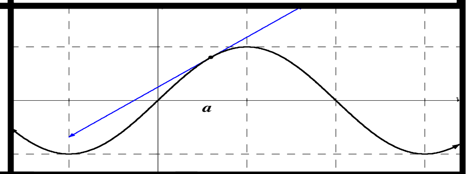
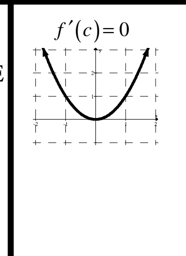
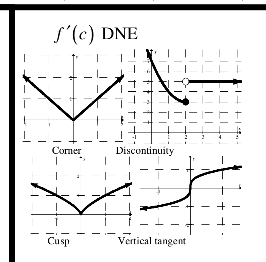
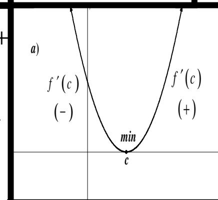
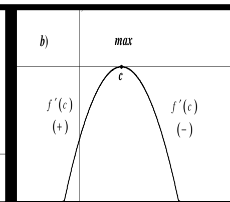
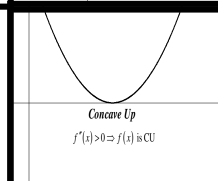
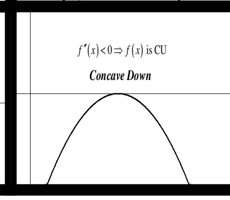
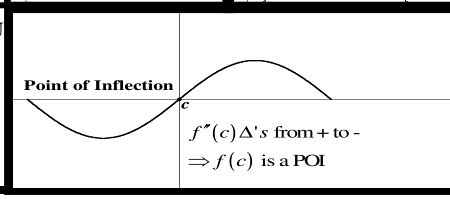
Intermediate Value Theorem: If f is cont on $[a,b]$ and $M\in(f(a),f(b))$

$\Rightarrow$ there exists a $c\in(a,b)$ such that $f(c)=M$

Average Value: $\text{av}(f)=f_{ave}=\frac{1}{b-a}\int_a^b f(x)dx$

Vol of rev $=\pi\int_a^b\left\{\left[f_{top}(x)-a\right]^2-\left[f_{bot}(x)-a\right]^2\right\}dx$ if $f_{top}>f_{bot}>a$

$=\pi\int_a^b\left\{\left[a-f_{bot}(x)\right]^2-\left[a-f_{top}(x)\right]^2\right\}dx$ if $a>f_{top}>f_{bot}$

Term	Verbal Description	Symbolic	Graphical
1. Derivative of f at a:	The instantaneous rate of change of the function at a or the slope of the tangent line at a	$f'(a)=\left.\frac{df}{dx}\right\|_{x=a} = \lim_{x\to z}\frac{f(a+h)-f(a)}{h}$	
2. Critical Number c	A number c in an open (a,b) interval where the derivative is zero or does not exist	$c\in(a,b)$ where $f'(c)=0$ or $f'(c)$ DNE	 
3. First Derivative Test	a) If the first derivative changes from negative to positive at c then the function has a relative minimum at c b) If the first derivative changes from positive to negative at c then the function has a relative maximum at c	a) If $f'(c)$ Δ's from $-$ to $+$ $\Rightarrow f'(c)$ is a min b) If $f'(c)$ Δ's from $+$ to $-$ $\Rightarrow f'(c)$ is a max	 
4. Concavity Test	a) If the second derivative is positive on an interval I then the function is Concave Up on I b) If the second derivative is negative on an interval I the function is Councave down on I	a) If $f''(c)>0$ on I $\Rightarrow f(x)$ is CU on I b) If $f''(c)<0$ on I $\Rightarrow f(x)$ is CD on I	 
5. Point of Inflection at c	***f*: Is a point where the concavity changes of *f*** ***f'*: Is a point where *f'* changes from increasing to decreasing or decreasing to increasing** ***f''*: Is a point where *f''* changes from positive to negative or negative to positive**	f Δ's from CU to CD or CD to CU f' Δ's from ↗ to ↘ or ↘ to ↗ $f''(x)$ Δ's from + to − or − to	

Motion definitions and Equations

6. Displacement: A Vector quantity that represents the net change in position $s(t)=x(b)-x(a)=\int_a^b v(t)$

7. Distance: A scalar quantity that represents total movement regardless of sign $d(t)=\|x(b)-x(a)\|=\int_a^b \|v(t)\|dt$

8. Velocity: A Vector quantity that represents the rate of change of position $v(t)=s'(t)$

9. Speed: A scalar quantity that represents the rate of covering distance Speed $=\|v(t)\|$

10. Acceleration: A vector quantity that represents the rate of change of velocity $a(t)=v'(t)=s''(t)$

11. Given initial position $s(a)=C$ the final position is given by $s(b)=s(a)+\int_a^b s'(t)\,dt$

Reciprocal

$\sin x=\frac{1}{\csc x}$ $\csc x=\frac{1}{\sin x}$

$\cos x=\frac{1}{\sec x}$ $\sec x=\frac{1}{\cos x}$

$\tan x=\frac{1}{\cot x}$ $\cot x=\frac{1}{\tan x}$

Quotient

$\tan x=\frac{\sin x}{\cos x}$

$\cot x=\frac{\cos x}{\sin x}$

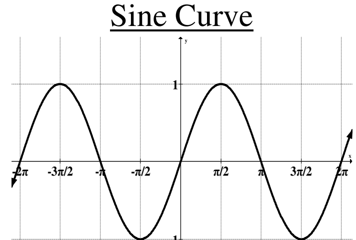
Pythagorean

$\sin^2 x+\cos^2 x=1$

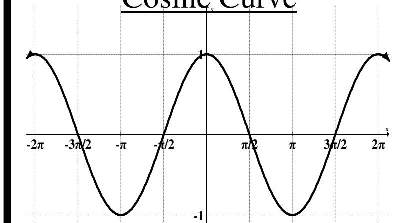
$\tan^2 x+1=\sec^2 x$

$\cot^2 x+1=\csc^2 x$

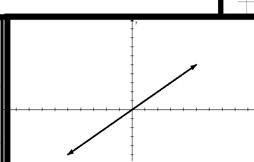
Sine Curve



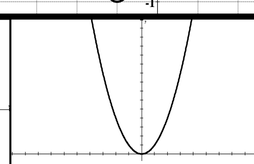
Cosine Curve



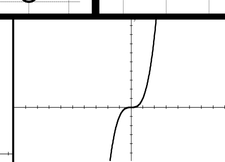
	0	$\pi/6$ (30°)	$\pi/4$ (45°)	$\pi/3$ (60°)	$\pi/2$ (90°)
sin x	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
cos x	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
tan x	0	$1/\sqrt{3}$	1	$\sqrt{3}$	Und.
csc x	Und.	2	$2/\sqrt{2}$	$2/\sqrt{3}$	1
sec x	1	$2/\sqrt{3}$	$2/\sqrt{2}$	2	Und.
cot x	Und.	$2/\sqrt{3}$	1	$1/\sqrt{3}$	0



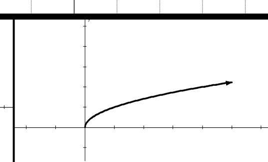
Linear $y=x$



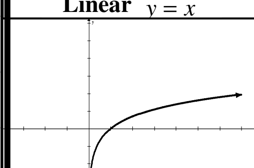
quadratic $y=x^2$



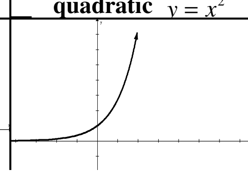
Cubic $y=x^3$



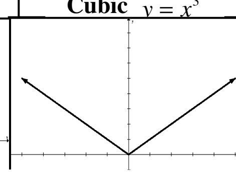
Radical $y=\sqrt{x}$



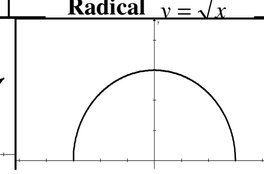
Logarithmic $y=\ln x$



Exponential $y=e^x$



Absolute value $y=\|x\|$



circular $y=\sqrt{9-x^2}$

A hemispherical bowl is being drained of water. The radius is modelled by a twice differentiable function R of time t. For 0 = t = 11 the water level is falling.
The table below gives values of R'(t) of t on 0 = t = 11 and R(7)=10

Note: The volume of a hemisphere is given by $V = \frac{2}{3}\pi r^3$

1. Approximate $R''(4.5)$. Show the computations that lead to your answer. What is the unit of your answer.
2. Estimate the radius of the water in the bowl when $t = 6.3$, using the tangent line approximation at $t = 6$. Is this an upper of lower estimate? Give a reason for your answer.
3. Use a left Riemann sum with four sub intervals to find $\int_0^{11} R'(t)\,dt$. State the meaning of your answer in the context of the problem with accurate units. Is this an over or under estimate?
4. Find $\int_0^{11} R''(t)$ dt show work state what your answer means with the accurate units.
5. Use a midpoint sum with two sub intervals to approximate $\frac{1}{11}\int_0^{11} R'(t)\,dt$. Using correct units explain what $\frac{1}{11}\int_0^{11} R'(t)\,dt$. means in the context of the problem.
6. Find the rate of change of volume at $t = 6$.
7. The Rate at which water leaves the bowl from $11 \le t \le 13$ is given by $J(t) = -\frac{1}{2}(t-11)^3 + 4$. Find the rate at which water is leaving the bowl when $t = 12$.

A hemispherical bowl is being drained of water. The radius is modelled by a twice differentiable function R of time t. For $0 \leq t \leq 11$ the water level is falling.

The table below gives values of the rate of change of radius $R'(t)$ of t on $0 \leq t \leq 11$ and $R(6) = 10$

Note: The volume of a hemisphere is given by $V = \frac{2}{3}\pi r^3$

t (mins)	0	3	6	8.5	11
$R'(t)$ m / min	10	8	6.5	5	4

1. Approximate $R''(4.5)$. Show the computations that lead to your answer. What is the unit of your answer.

2. Estimate the radius of the water in the bowl when $t = 6.3$, using the tangent line approximation at $t = 6$. Is this an upper of lower estimate? Give a reason for your answer.

3. Use a left Riemann sum with four sub intervals to find $\int_0^{11} R'(t)\,dt$. State the meaning of your answer in the context of the problem with accurate units. Is this an over or under estimate?

4. Find $\int_0^{11} R''(t)$ dt show work state what your answer means with the accurate units.

5. Use a midpoint sum with two sub intervals to approximate $\frac{1}{11}\int_0^{11} R'(t)\,dt$. Using correct units explain what $\frac{1}{11}\int_0^{11} R'(t)\,dt$. means in the context of the problem.

6. Find the rate of change of volume at $t = 6$.

7. The derivative of the rate at which water leaves the bowl from $11 \leq t \leq 13$ is given by $R''(t) = -\frac{1}{2}(t-11)^3 + 4$. Find the rate at which water is leaving the bowl when $t = 12$.

8. Does $R''(t)$ ever attain a value of $-\frac{2}{3}$ on $(0,3)$ Explain your reasoning.